Analysis of multi-story multi-bay RC plane frame

Input data

Number of stories - $n_{st} = 5$,   Number of bays - $n_b = 3$

Story height - $h_{st} = 2.85\ \mathrm{m}$,   Bay length- $l_b = 4\ \mathrm{m}$

Slab thickness - $h_{pl} = 18\ \mathrm{cm}$

Cross section of columns - $b_c = 25\ \mathrm{cm}$, $h_c = 60\ \mathrm{cm}$

Cross section of beams - $b_b = 25\ \mathrm{cm}$, $h_b = 40\ \mathrm{cm}$

Joint coordinates - $n_J = 24$

$\vec{x}_J = [0\ \ 4\ \ 8\ \ 12\ \ 0\ \ 4\ \ 8\ \ 12\ \ 0\ \ 4\ \ldots 12]\ \mathrm{m}$, $\vec{y}_J = [0\ \ 0\ \ 0\ \ 0\ \ 2.85\ \ 2.85\ \ 2.85\ \ 2.85\ \ 5.7\ \ 5.7 \ldots 14.25]\ \mathrm{m}$

Elements - [J1; J2] - $n_E = 35$

$$\mathbf{transp}(e_J) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & \cdots & 23 \\ 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & \cdots & 24 \end{bmatrix}$$

Element endpoint coordinates

$x_1(e) = \vec{x}_{J.e_{J_{e,1}}}$, $y_1(e) = \vec{y}_{J.e_{J_{e,1}}}$, $x_2(e) = \vec{x}_{J.e_{J_{e,2}}}$, $y_2(e) = \vec{y}_{J.e_{J_{e,2}}}$

Element length - $l(e) = \sqrt{\left(x_2(e) - x_1(e)\right)^2 + \left(y_2(e) - y_1(e)\right)^2}$

Element direction - $c(e) = \frac{x_2(e) - x_1(e)}{l(e)}$, $s(e) = \frac{y_2(e) - y_1(e)}{l(e)}$

Transformation matrix

Diagonal 3x3 block - $t(e) = \mathbf{hp}([c(e); s(e); 0 \mid -s(e); c(e); 0 \mid 0; 0; 1])$

Generation of the full transformation matrix

$T(e) = \mathbf{add}(t(e); \mathbf{add}(t(e); \mathbf{matrix}(6; 6); 1; 1); 4; 4)$

Supports - $n_c = 4$

$$c = \begin{bmatrix} 1 & 10^{20}\ \mathrm{kN/m} & 10^{20}\ \mathrm{kN/m} & 0\ \mathrm{kNm} \\ 2 & 10^{20}\ \mathrm{kN/m} & 10^{20}\ \mathrm{kN/m} & 0\ \mathrm{kNm} \\ 3 & 10^{20}\ \mathrm{kN/m} & 10^{20}\ \mathrm{kN/m} & 0\ \mathrm{kNm} \\ 4 & 10^{20}\ \mathrm{kN/m} & 10^{20}\ \mathrm{kN/m} & 0\ \mathrm{kNm} \end{bmatrix}$$

Unit weights of building materials

- concrete - $\gamma_c = 25\ \mathrm{kN/m^3}$
- finishes - $\gamma_{fin} = 18\ \mathrm{kN/m^3}$
- plaster/render - $\gamma_{pla} = 16\ \mathrm{kN/m^3}$
- screed - $\gamma_{scr} = 21\ \mathrm{kN/m^3}$
- brickwork - $\gamma_{bw} = 16\ \mathrm{kN/m^3}$
- insulation - $\gamma_{ins} = 0.5\ \mathrm{kN/m^3}$

Loads

Total halfwidth of adjacent plate spans - $a = \frac{5\,\mathrm{m}}{2} = 2.5\,\mathrm{m}$

Self weight

Plate - $g_{pl} = h_{pl} \cdot a \cdot \gamma_c = 11.25\,\mathrm{kN/m}$

Beam - $g_b = b_b \cdot \left(h_b - h_{pl}\right) \cdot \gamma_c = 1.38\,\mathrm{kN/m}$

Total for beam - $sw = g_{pl} + g_b = 12.62\,\mathrm{kN/m}$

Column - $g_c = b_c \cdot h_c \cdot \gamma_c = 3.75\,\mathrm{kN/m}$

Dead loads

Screed - $g_{scr} = 8\,\mathrm{cm} \cdot a \cdot \gamma_{scr} = 4.2\,\mathrm{kN/m}$

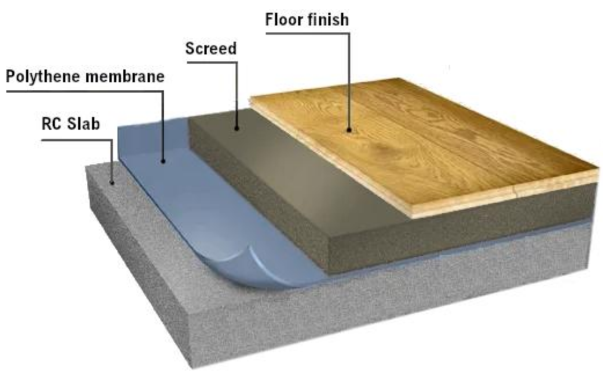
Finishes - $g_{fin} = 2\,\mathrm{cm} \cdot a \cdot \gamma_{fin} = 0.9\,\mathrm{kN/m}$

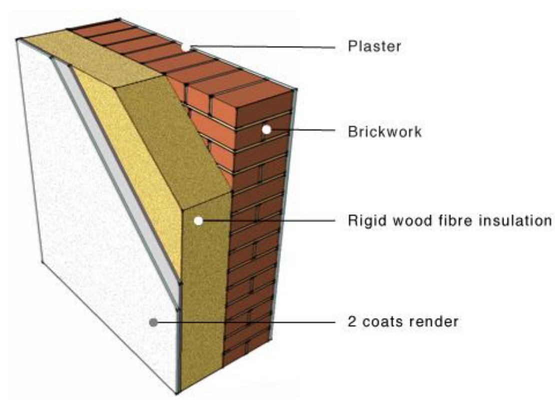
Plaster ceiling - $g_{pls} = 2\,\mathrm{cm} \cdot a \cdot \gamma_{pla} = 0.8\,\mathrm{kN/m}$

Brick wall - $g_{bw} = 25\,\mathrm{cm} \cdot (h_{st} - h_b) \cdot \gamma_{bw} = 9.8\,\mathrm{kN/m}$

Wall insulation - $g_{ins} = 15\,\mathrm{cm} \cdot h_{st} \cdot \gamma_{ins} = 0.214\,\mathrm{kN/m}$

Wall plaster/render - $g_{plw} = 2 \cdot 2\,\mathrm{cm} \cdot h_{st} \cdot \gamma_{pla} = 1.82\,\mathrm{kN/m}$





Total dead load

$dl = g_{scr} + g_{fin} + g_{pls} + g_{bw} + g_{ins} + g_{plw} = 17.74\,\mathrm{kN/m}$

Live load - $ll = a \cdot 2\,\frac{\mathrm{kN}}{\mathrm{m}^2} = 5\,\mathrm{kN/m}$

Total load

On beams - $p_b = (sw + dl) \cdot 1.35 + ll \cdot 1.5 = 48.49\,\mathrm{kN/m}$

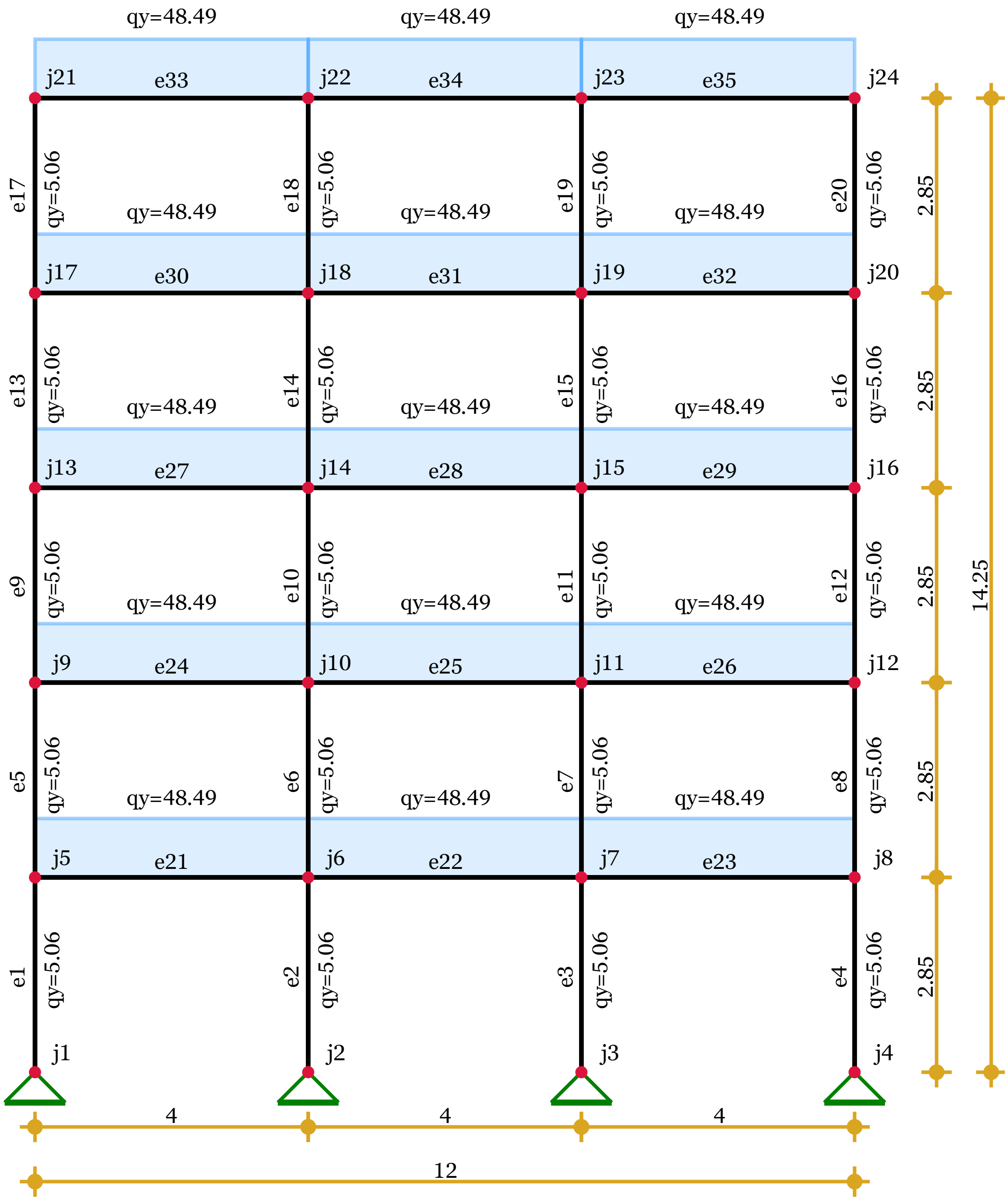
On columns - $p_c = g_c \cdot 1.35 = 5.06\,\mathrm{kN/m}$

Load values on elements

$\vec{q}_x = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \ldots\ \ 0]\,\mathrm{kN/m}$

$\vec{q}_y = [-5.06\ \ -5.06\ \ -5.06\ \ -5.06\ \ -5.06\ \ -5.06\ \ -5.06\ \ -5.06\ \ -5.06\ \ -5.06\ \ \ldots\ \ -48.49]\,\mathrm{kN/m}$

Scheme of the structure



Materials

Modules of elasticity - $\vec{E} = \mathbf{hp}([35\,\mathrm{GPa}]) = [35]\,\mathrm{GPa}$

Poisson coefficients - $\vec{\nu} = \mathbf{hp}([0.2]) = [0.2]$

Shear modules - $\vec{G} = \frac{\vec{E}}{2\cdot(1+\vec{\nu})} = [14.58]\,\mathrm{GPa}$

Assignments on elements

$\vec{e}_M = \mathbf{fill}\left(\mathbf{vector}_{hp}(n_E); 1\right) = [1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ \ldots\ \ 1]$

Cross sections

Calculation of effective flange width

$l_0 = 0.85 \cdot l_b = 3.4\,\mathrm{m}$

$b_{eff} = b_b + \mathbf{min}(0.2 \cdot a + 0.1 \cdot l_0; 0.2 \cdot l_0) = 93\,\mathrm{cm}$

Section 1 - $\vec{b}_1 = b_c = 25\,\mathrm{cm}$, $\vec{h}_1 = h_c = 60\,\mathrm{cm}$ - columns

Section 2 - $\vec{b}_2 = b_b = 25\,\mathrm{cm}$, $\vec{h}_2 = h_b = 40\,\mathrm{cm}$ - beams

$\vec{b}_{f_.} = b_{eff} = 93\,\mathrm{cm}$, $\vec{h}_{f_.} = h_{pl} = 18\,\mathrm{cm}$

Cross section properties

Area

Web - $\vec{A}_w = \vec{b} \odot \vec{h} = [1500\ \ 1000]\,\mathrm{cm}^2$

Flange - $\vec{A}_f = \left(\vec{b}_f - \vec{b}\right) \odot \vec{h}_f = [0\ \ 1224]\,\mathrm{cm}^2$

Total - $\vec{A} = \vec{A}_w + \vec{A}_f = [1500\ \ 2224]\,\mathrm{cm}^2$

First moment of area - $\vec{S} = \frac{\vec{A}_w \odot \vec{h}}{2} + \vec{A}_f \odot \left(\vec{h} - \frac{\vec{h}_f}{2}\right) = [45000\ \ 57944]\,\mathrm{cm}^3$

Geometrical center - $\vec{z}_c = \frac{\vec{S}}{\vec{A}} = [300\ \ 260.54]\,\mathrm{mm}$

Second moment of area

Web - $\vec{I}_w = \vec{A}_w \odot \left(\frac{\vec{h}^{\odot 2}}{12} + \left(\vec{z}_c - \frac{\vec{h}}{2}\right)^{\odot 2}\right) = [450000\ \ 169984]\,\mathrm{cm}^4$

Flange - $\vec{I}_f = \vec{A}_f \odot \left(\frac{\vec{h}_f^{\ \odot 2}}{12} + \left(\vec{h} - \vec{z}_c - \frac{\vec{h}_f}{2}\right)^{\odot 2}\right) = [0\ \ 62991.1]\,\mathrm{cm}^4$

Total - $\vec{I} = \vec{I}_w + \vec{I}_f = [450000\ \ 232975]\,\mathrm{cm}^4$

Shear area - $\vec{A}_s = \frac{\vec{A}}{1.2} = [1250\ \ 1853.33]\,\mathrm{cm}^2$

Assignment on elements

Columns - $\vec{e}_{SC} = \mathbf{fill}\left(\mathbf{vector}_{hp}(n_{CE}); 1\right) = [1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ \ldots\ \ 1]$

Beams - $\vec{e}_{SB} = \mathbf{fill}\left(\mathbf{vector}_{hp}(n_{BE}); 2\right) = [2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ \ldots\ \ 2]$

All- $\vec{e}_S = \mathbf{hp}([\vec{e}_{SC}; \vec{e}_{SB}]) = [1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ \ldots\ \ 2]$

Element stiffness matrix

Elastic properties for element "e"

$$EA(e) = \vec{E}_{\vec{e}_{M.e}} \cdot \vec{A}_{e_{S.e}}\ ,\ EI(e) = \vec{E}_{\vec{e}_{M.e}} \cdot \vec{I}_{e_{S.e}}\ ,\ GA_s(e) = \vec{G}_{\vec{e}_{M.e}} \cdot \vec{A}_{s.e_{S.e}}$$

$$k_s(e) = \frac{12 \cdot EI(e)}{GA_s(e) \cdot l(e)^2}\ ,\ \alpha(e) = \frac{EA(e)}{l(e)}\ ,\ \beta(e) = \frac{EI(e)}{l(e)^3 \cdot (1+k_s(e))}$$

Stiffness matrix coefficients for element "e"

$$k_{11}(e) = \alpha(e) \cdot \frac{m}{kN}\ ,\ k_{22}(e) = 12 \cdot \beta(e) \cdot \frac{m}{kN}\ ,\ k_{23}(e) = 6 \cdot \beta(e) \cdot l(e) \cdot \frac{1}{kN}$$

$$k_{33}(e) = \left(4 + k_s(e)\right) \cdot \beta(e) \cdot l(e)^2 \cdot \frac{1}{kNm}\ ,\ k_{36}(e) = \left(2 - k_s(e)\right) \cdot \beta(e) \cdot l(e)^2 \cdot \frac{1}{kNm}$$

Assembling the 3x3 stiffness matrix blocks for element "e"

$$k_{ii}(e) = \mathbf{hp}([k_{11}(e)\ |\ 0; k_{22}(e); k_{23}(e)\ |\ 0; k_{23}(e); k_{33}(e)])$$

$$k_{ij}(e) = \mathbf{hp}([-k_{11}(e)\ |\ 0; -k_{22}(e); k_{23}(e)\ |\ 0; -k_{23}(e); k_{36}(e)])$$

$$k_{ji}(e) = \mathbf{transp}\left(k_{ij}(e)\right)$$

$$k_{jj}(e) = \mathbf{hp}([k_{11}(e)\ |\ 0; k_{22}(e); -k_{23}(e)\ |\ 0; -k_{23}(e); k_{33}(e)])$$

Full element stiffness matrix

$$k_E(e) = \mathbf{stack}\left(\mathbf{augment}\left(k_{ii}(e); k_{ij}(e)\right); \mathbf{augment}\left(k_{ji}(e); k_{jj}(e)\right)\right)$$

Stiffness matrices obtained in local coordinates

$$k_E(1) = \begin{bmatrix} 1842105 & 0 & 0 & -1842105 & 0 & 0 \\ 0 & 72402.7 & 103174 & 0 & -72402.7 & 103174 \\ 0 & 103174 & 202286 & 0 & -103174 & 91759.5 \\ -1842105 & 0 & 0 & 1842105 & 0 & 0 \\ 0 & -72402.7 & -103174 & 0 & 72402.7 & -103174 \\ 0 & 103174 & 91759.5 & 0 & -103174 & 202286 \end{bmatrix}$$

$$n_{b1} = n_{CE} + 1 = 21$$

$$k_E(n_{b1}) = \begin{bmatrix} 1946000 & 0 & 0 & -1946000 & 0 & 0 \\ 0 & 14950.7 & 29901.4 & 0 & -14950.7 & 29901.4 \\ 0 & 29901.4 & 80188 & 0 & -29901.4 & 39417.4 \\ -1946000 & 0 & 0 & 1946000 & 0 & 0 \\ 0 & -14950.7 & -29901.4 & 0 & 14950.7 & -29901.4 \\ 0 & 29901.4 & 39417.4 & 0 & -29901.4 & 80188 \end{bmatrix}$$

Stiffness matrices obtained in global coordinates

$$\mathbf{transp}(T(1)) \cdot k_E(1) \cdot T(1) = \begin{bmatrix} 72402.7 & 0 & -103174 & -72402.7 & 0 & -103174 \\ 0 & 1842105 & 0 & 0 & -1842105 & 0 \\ -103174 & 0 & 202286 & 103174 & 0 & 91759.5 \\ -72402.7 & 0 & 103174 & 72402.7 & 0 & 103174 \\ 0 & -1842105 & 0 & 0 & 1842105 & 0 \\ -103174 & 0 & 91759.5 & 103174 & 0 & 202286 \end{bmatrix}$$

$$\text{transp}\left(T\left(n_{b1}\right)\right) \cdot k_E\left(n_{b1}\right) \cdot T\left(n_{b1}\right) = \begin{bmatrix} 1946000 & 0 & 0 & -1946000 & 0 & 0 \\ 0 & 14950.7 & 29901.4 & 0 & -14950.7 & 29901.4 \\ 0 & 29901.4 & 80188 & 0 & -29901.4 & 39417.4 \\ -1946000 & 0 & 0 & 1946000 & 0 & 0 \\ 0 & -14950.7 & -29901.4 & 0 & 14950.7 & -29901.4 \\ 0 & 29901.4 & 39417.4 & 0 & -29901.4 & 80188 \end{bmatrix}$$

Global stiffness matrix

$$K = \begin{bmatrix} 10^{20} & 0 & -103174 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 10^{20} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ -103174 & 0 & 202286 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 10^{20} & 0 & -103174 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 10^{20} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & -103174 & 0 & 202286 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10^{20} & 0 & -103174 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^{20} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -103174 & 0 & 202286 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^{20} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 282474 \end{bmatrix}$$

Element load vector

Lateral load in local CS - $q_E(e) = -\vec{q}_{x.e} \cdot s(e) + \vec{q}_{y.e} \cdot c(e)$

Axial load in local CS - $n_E(e) = \vec{q}_{x.e} \cdot c(e) + \vec{q}_{y.e} \cdot s(e)$

Equivalent loads at element endpoints

$F_{Ex}(e) = \frac{\vec{q}_{x.e} \cdot l(e)}{2} \cdot \frac{1}{kN}$, $F_{Ey}(e) = \frac{\vec{q}_{y.e} \cdot l(e)}{2} \cdot \frac{1}{kN}$, $M_E(e) = \frac{q_E(e) \cdot l(e)^2}{12} \cdot \frac{1}{kNm}$

Load vector - $F_E(e) = \text{hp}\left(\left[F_{Ex}(e); F_{Ey}(e); M_E(e); F_{Ex}(e); F_{Ey}(e); -M_E(e)\right]\right)$

Global load vector

$\vec{F} = [0\ \ -7.21\ \ 0\ \ 0\ \ -7.21\ \ 0\ \ 0\ \ -7.21\ \ 0\ \ 0\ \ \ldots\ \ 64.65]$

Results

Solution of the system of equations by PCG method

$\vec{Z} = \text{slsolve}\left(K; \vec{F}\right)$

$= [0\ \ -5.72\times10^{-18}\ \ 7.06\times10^{-5}\ \ 0\ \ -1.03\times10^{-17}\ \ 3.03\times10^{-6}\ \ 0\ \ -1.03\times10^{-17}\ \ -3.03\times10^{-6}\ \ 0\ \ \ldots\ \ 0.000283]$

Joint displacements

$z_J(j) = \text{slice}\left(\vec{Z}; 3 \cdot j - 2; 3 \cdot j\right)$ $\qquad$ $z(j) = \text{round}\left(\frac{z_J(j)}{\delta z}\right) \cdot \delta z \cdot 1000 \cdot [mm; mm; 1]$

Support reactions

$r(i) = \text{row}(c; i)$, $j(i) = \text{take}\left(1; r(i)\right)$

$R(i) = -z_J\left(j(i)\right) \cdot [m; m; 1] \cdot \text{last}\left(r(i); 3\right)$

Joint **J1 -** $[8.19\,\mathrm{kN}\ \ 571.77\,\mathrm{kN}\ \ 0\,\mathrm{kNm}]$

Joint **J2 -** $[0.174\,\mathrm{kN}\ \ 1027.2\,\mathrm{kN}\ \ 0\,\mathrm{kNm}]$

Joint **J3 -** $[-0.174\,\mathrm{kN}\ \ 1027.2\,\mathrm{kN}\ \ 0\,\mathrm{kNm}]$

Joint **J4 -** $[-8.19\,\mathrm{kN}\ \ 571.77\,\mathrm{kN}\ \ 0\,\mathrm{kNm}]$

Element end forces

$$z_E(e) = \mathbf{hp}\left(\left[z_J\left(e_{J_{e,1}}\right); z_J\left(e_{J_{e,2}}\right)\right]\right)$$

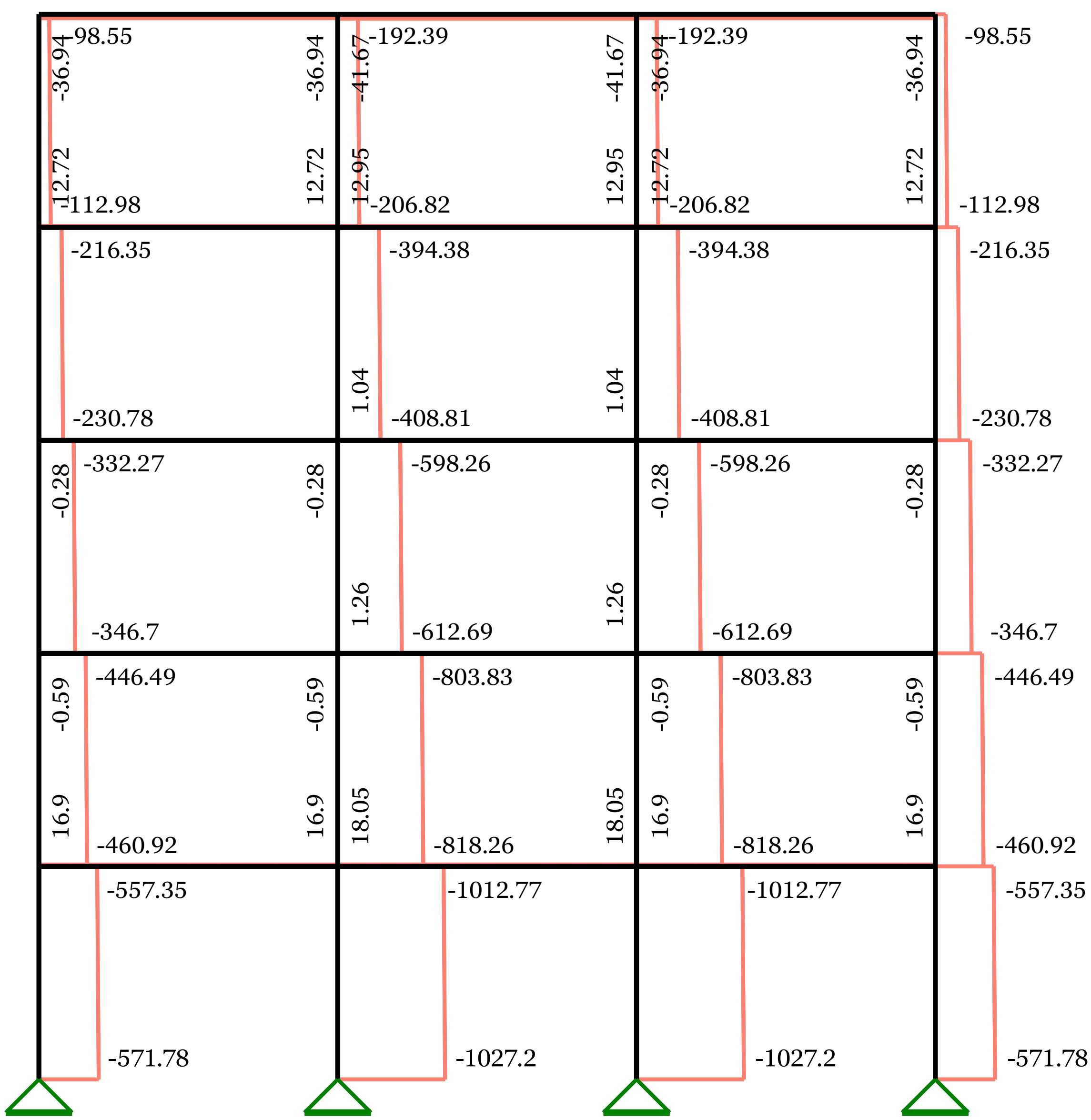
$$R_E(e) = \mathbf{col}\left(k_E(e)\cdot T(e)\cdot z_E(e) - T(e)\cdot F_E(e); 1\right)\cdot[1;1;m;1;1;m]\cdot kN$$

Element internal forces

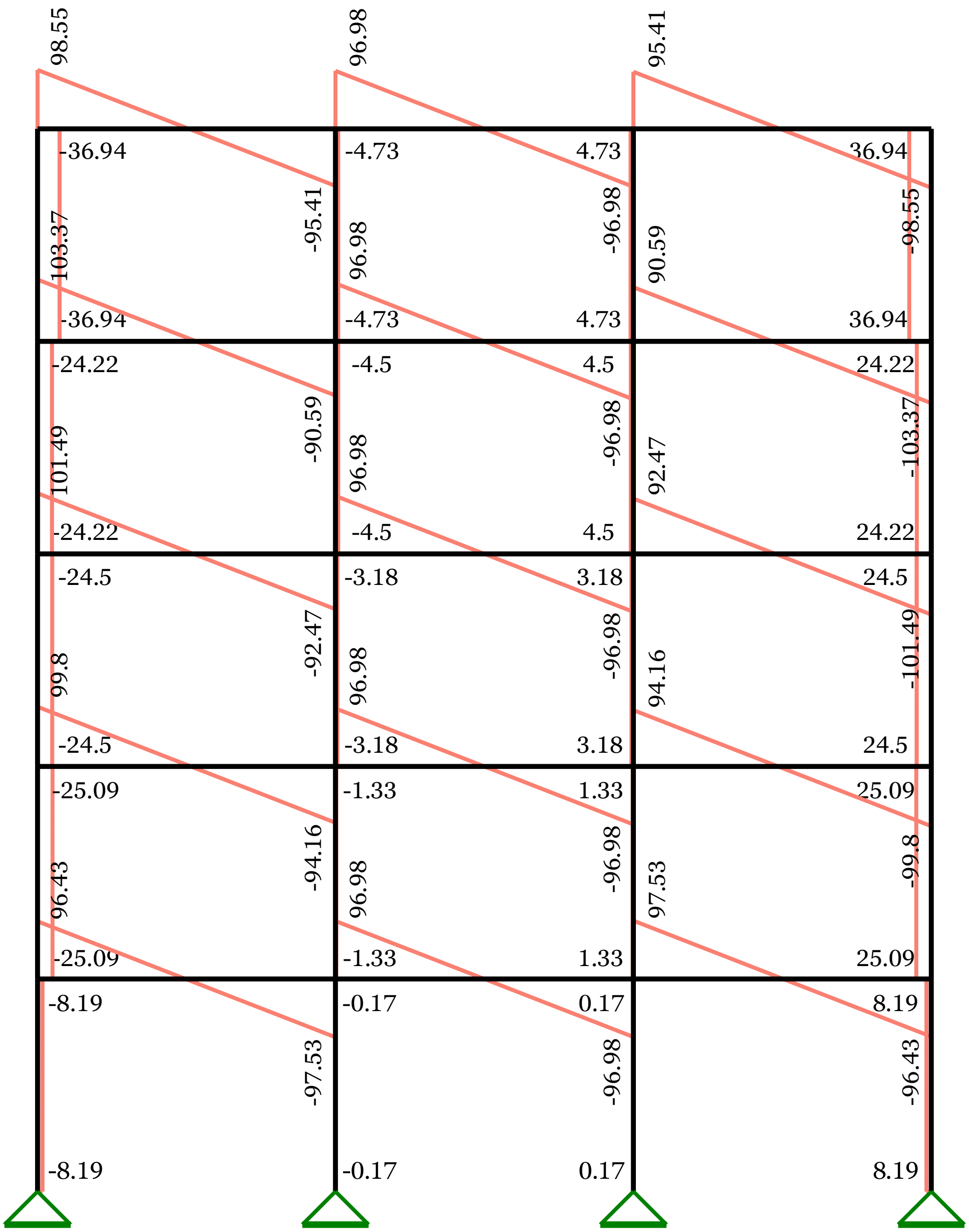
$$N(e;x) = \text{-}\mathbf{take}\left(1; R_E(e)\right) - n_E(e)\cdot x\ ,\ \ Q(e;x) = \mathbf{take}\left(2; R_E(e)\right) + q_E(e)\cdot x$$

$$M(e;x) = \text{-}\mathbf{take}\left(3; R_E(e)\right) + \mathbf{take}\left(2; R_E(e)\right)\cdot x + \frac{q_E(e)\cdot x^2}{2}$$

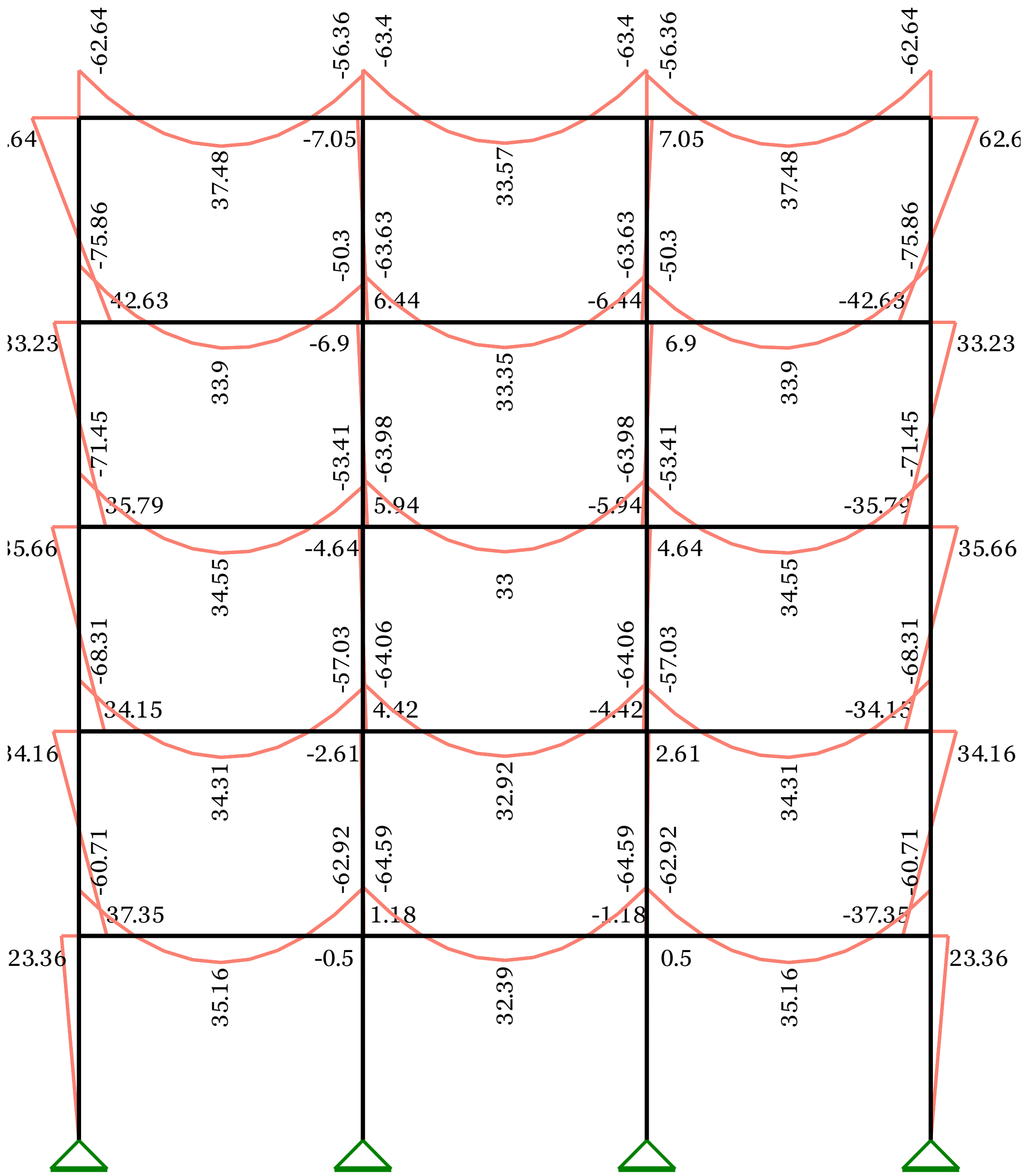
Axial forces diagram, kN



Shear forces diagram, kN



Bending moments diagram, kNm



Deformed shape

Shape function in relative coordinates ξ = x/l (with account to shear deflections)

$$\Phi_1(e;\xi) = \frac{1}{1+k_s(e)} \cdot \left(1 + k_s(e) - k_s(e)\cdot\xi - 3\cdot\xi^2 + 2\cdot\xi^3\right)$$

$$\Phi_2(e;\xi) = \frac{\xi \cdot l(e) \cdot m^{-1}}{1+k_s(e)} \cdot \left(1 + \frac{k_s(e)}{2} - \left(2 + \frac{k_s(e)}{2}\right)\cdot\xi + \xi^2\right)$$

$$\Phi_3(e;\xi) = \frac{\xi}{1+k_s(e)} \cdot \left(k_s(e) + 3\cdot\xi - 2\cdot\xi^2\right)$$

$$\Phi_4(e;\xi) = \frac{\xi \cdot l(e) \cdot m^{-1}}{1+k_s(e)} \cdot \left(\frac{-k_s(e)}{2} - \left(1 - \frac{k_s(e)}{2}\right)\cdot\xi + \xi^2\right)$$

Element endpoint displacements and rotations

$$z_{E,loc}(e) = T(e) \cdot z_E(e)$$

$$u_1(e) = \textbf{take}\left(1; z_{E,loc}(e)\right), v_1(e) = \textbf{take}\left(2; z_{E,loc}(e)\right), \varphi_1(e) = \textbf{take}\left(3; z_{E,loc}(e)\right)$$

$$u_2(e) = \textbf{take}\left(4; z_{E,loc}(e)\right), v_2(e) = \textbf{take}\left(5; z_{E,loc}(e)\right), \varphi_2(e) = \textbf{take}\left(6; z_{E,loc}(e)\right)$$

Displacement functions (with account for intermediate loads)

$$u(e; \xi) = u_1(e) \cdot (1 - \xi) + u_2(e) \cdot \xi + \frac{n_E(e) \cdot \mathrm{m}}{EA(e)} \cdot \xi \cdot (1 - \xi)$$

$$v(e; \xi) = v_1(e) \cdot \Phi_1(e; \xi) + \varphi_1(e) \cdot \Phi_2(e; \xi) + v_2(e) \cdot \Phi_3(e; \xi) + \varphi_2(e) \cdot \Phi_4(e; \xi) + \frac{q_E(e) \cdot l(e)^4}{24 \cdot EI(e)} \cdot \frac{\xi^2 \cdot (1 - \xi)^2}{\mathrm{m}} + \frac{q_E(e) \cdot l(e)^2}{2 \cdot GA_s(e)} \cdot \frac{\xi \cdot (1 - \xi)}{\mathrm{m}}$$

Deformed shape, mm